Oedometer Consolidation Test Analysis by Nonlinear Regression

ABSTRACT: A numerical method based on least squares nonlinear regression for the evaluation of the consolidation parameters of soils from consolidation tests is presented. A model which includes the initial compression, the primary consolidation, and the secondary compression is used in the regression. This approach allows the resulting regression curve to better fit the experimental data. The method takes the settlement-time readings from the oedometer step-loading consolidation test and calculates automatically the magnitudes of the coefficients of consolidation and of secondary compression. The performance of the proposed method is accessed through consolidation tests executed on four different clay soils, which are analyzed by nonlinear regression and by the usual graphical methods. It is concluded that the proposed method gives results that are close to those obtained by the standard methods of analysis.

KEYWORDS: consolidation test, nonlinear regression, least squares, coefficient of consolidation, secondary compression

Introduction

The oedometer step-loading consolidation test is one of the most widely used tests in the soil mechanics laboratory. Introduced by Terzaghi as an experimental support for his one-dimensional consolidation theory (Terzaghi 1943), the oedometer test has remained essentially unchanged since then.

The main objective of the consolidation test is to access the consolidation characteristics of a soil from the measured settlement-time curve. From the consolidation theory, those characteristics are expressed by the soil coefficient of consolidation $c_v$ and total consolidation settlement $\delta_{100}$.

The evaluation of $c_v$ and $\delta_{100}$ from the measured consolidation curve is generally performed by hand-draft curve fitting, being the Taylor’s $\delta \times \sqrt{t}$ method (Taylor 1948) and the Casagrande’s $\delta \times \log t$ method (Casagrande and Fadum 1940), the most used. Those are regarded as standard methods of $c_v$ evaluation. Other methods were developed later for this same purpose, such as the rectangular hyperbola method (Shidharan and Prakash 1985) and the early stage log $t$ method (Robinson and Allam 1996). But all the curve fitting methods above require graphical constructions that introduce undesirable subjective interpretations in the process.

The various curve fitting methods lead, in general, to results that are different from each other because each one focuses on different portions of the consolidation curve. The main cause of these discrepancies is the experimental consolidation curve departure from theory, which is mainly caused by:

- There is an initial settlement $\delta_0$ just after the load application, which is due to incomplete sample saturation, confining ring expansion, and deformation of the loading apparatus.
- The settlement continues after the theoretical end of consolidation in a process known as secondary compression.

The initial settlement is easily treated by the various graphical methods and poses no special difficulties to the analysis. On the other hand, secondary compression interpretation is much more challenging for there is no well established theoretical model for it. Even the point in time when secondary compression may first be detected is subject to controversy (see, e.g., Robinson 2003).

Added to those discrepancies, the observed consolidation curve may also differ from the theoretical model due to the nonlinear behavior of the soil compressibility during the load increment and even due to limitations of the test equipment and instrumentation.

It is worth noting that if the soil behavior during the consolidation followed Terzaghi’s consolidation theory then all those fitting methods would lead to the very same results. But the deviations from theory make the result dependent on the particular aspect of the theoretical behavior each fitting method arbitrarily takes into account. In this way, a less arbitrary procedure, with a sounder statistical base, like those based on least squares regression, is desirable.

The automatic interpretation of the consolidation curve through least squares regression was implemented in recent works (Robinson and Allam 1998; Chan 2003; Day and Morris 2006). The proposed method adds to those the inclusion of secondary compression in the regression model. As a consequence, the resulting regression curve fits better to the final points of the experimental consolidation curve. Thus, the effects of the secondary compression on the resulting $c_v$ value can be controlled. Furthermore, the regression formulation developed here can be easily implemented in spreadsheet programs which are available in most soil mechanics laboratories. The main benefits of the proposed method are:

- The need for human intervention in the interpretation process is kept to a minimum. In this way, manual calculation errors are avoided.
- The interpretation process takes much less time, so the laboratory technicians can focus their attention on the test execution procedures.
- The values of the consolidation parameters obtained are essentially free of subjectivity, which makes correlations between them and other soil parameters more consistent.
The complete model resulting from the superposition of the three parts is expressed by:

\[ \delta(t) = \delta_0 + \delta_{100}U(T) + c_v \frac{H}{1 + e_0 \log_{10} \frac{t}{t_0}} \]  

where \( \delta_0 \), \( \delta_{100} \), \( c_v \), and \( H \) are model parameters. The model expressed by Eq 7 is linear with respect to \( \delta_0 \), \( \delta_{100} \), and \( c_v \), and nonlinear only with respect to \( c_v \). This fact allows for a specialized nonlinear least squares regression procedure.

The least squares procedure is expressed by the minimization:

\[ \min \sum_{i=1}^{N} [\delta(t_i) - \delta_i]^2 \]  

where \( (t_i, \delta_i) \) are the time-settlement pairs measured during the consolidation test and \( N \) is the number of readings in the load increment stage being analyzed.

Since the model is linear with respect to all but one parameter, the problem reduces, through the use of derivatives with respect to each parameter, to a system of three linear simultaneous equations coupled to one nonlinear equation. Then, the solution of the complete system of equations can be obtained using a simple \( 3 \times 3 \) system of linear equations solver coupled to a numerical root finding method. The proposed regression method can be implemented in common spreadsheet programs in a very straightforward way, either through macro-programming or entirely with spreadsheet built functions. In the last case, most spreadsheet programs do incorporate in their standard configuration a solver for systems of linear equations and also a root finding facility called “goal seek,” which can be used in the procedure. Derivation details for the proposed method are given in the Appendix.
As an alternative, one can use Eq 8 directly as input to a general minimization routine, also found in some spreadsheet programs. This approach has some drawbacks when compared to the procedure used here, though. An initial guess for all four model parameters is required when using a general minimization routine, whereas for the specialized procedure, only an initial guess for \( c_v \) is required. Moreover, general minimization routines are more ill-convergence prone. That is, the minimization procedure can fail to converge, or converge to unsatisfactory values. The proposed procedure is, by contrast, much more stable and robust, because the nonlinear minimization is performed in only one direction. In the other three directions the minimization is accomplished by the solution of a simple system of linear equations.

Consolidation Tests

In order to access the performance of the proposed regression method, a series of consolidation tests on different types of clay soils were executed. Three clay samples were used. The first one is a low plasticity, bluish-gray clay from a deposit near the city of Santa Gertrudes, in the State of São Paulo, Brazil. The second clay is a commercial bentonite sample, from the State of Paraiba, Brazil. The last sample is a highly organic, black, marine clay, from the city of Cubatão, State of São Paulo, Brazil. The main characteristics of those samples are given in Table 1, where \( G_s \) is the specific gravity of soil solids, \( LL \) is the liquid limit, and \( PI \) is the plasticity index.

Four remolded soil specimens for the consolidation tests were prepared with those three samples. The first one was prepared with Sample 1 only. The second and third specimens were prepared from mixtures of Sample 1 and Sample 2. Finally, Specimen 4 was prepared with Sample 3 only. The specimens’ composition is shown in Table 2.

The specimens, measuring 71.4 mm in diameter and 20 mm in height, were prepared for the tests by putting the soil mixture at a water content in the plastic range and then gently molding by hand a small block from which the specimen was cut with the test ring. In this way, very soft, saturated specimens for which the magnitude order of the coefficient of consolidation remained constant for all load increments could be molded. The specimens initial water content \( w_0 \) and void ratio \( e_0 \) are also shown in Table 2.

The consolidation cell uses a fixed-type ring setup with drainage on both top and bottom ends of the specimen. The consolidation tests were conducted in the standard way, with load increment ratio \( LIR = 1 \), with first load \( q_1 = 12.5 \text{ kPa} \) and last load \( q_8 = 1600 \text{ kPa} \).

![FIG. 1—Consolidation curve for load increment from 100 kPa to 200 kPa, for Specimen 1.](image1)

After each load increment, settlement measurements at times \( t \) in a geometric sequence with ratio \( t_i/t_{i-1} = \sqrt{2} \) were taken. For those measurements, a digital dial gage with 0.001 mm resolution, linked to a data logging program in a computer was used. Each load increment was left on the soil specimen for 24 hours, or until the secondary compression was clearly defined.

After the tests, the datasets were processed both by hand calculation and by the proposed regression method. The hand calculation was carried out independently by two experienced laboratory technicians; one of them using Taylor’s square root of time method and the other using Casagrande’s logarithm of time method.

Results and Discussion

The plots in Figs. 1–4 show the measured points along with the regression curves obtained with the proposed method, for the load increment from \( \sigma_1’ = 100 \text{ kPa} \) to \( \sigma_1’ = 200 \text{ kPa} \), for the four specimens. The agreement between the data points and the regression curves is excellent in all cases. The diversity in the curve characteristics for each soils is worth noting. Specimen 1 curve shows small deformability and high coefficient of consolidation \( c_v \). As the content of bentonite in the soil is increased in Specimens 2 and 3, the deformability increases and the magnitude of \( c_v \) decreases. The coefficient of secondary compression \( c_a \) also increases with the bentonite content.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Bluish-gray clay</th>
<th>Bentonite</th>
<th>Organic Clay</th>
<th>( w_0 )</th>
<th>( e_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100 %</td>
<td>0 %</td>
<td>0 %</td>
<td>36.9 %</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>75 %</td>
<td>25 %</td>
<td>0 %</td>
<td>63.4 %</td>
<td>1.72</td>
</tr>
<tr>
<td>3</td>
<td>50 %</td>
<td>50 %</td>
<td>0 %</td>
<td>79.6 %</td>
<td>2.23</td>
</tr>
<tr>
<td>4</td>
<td>0 %</td>
<td>0 %</td>
<td>100 %</td>
<td>177.1 %</td>
<td>3.33</td>
</tr>
</tbody>
</table>

![FIG. 2—Consolidation curve for load increment from 100 kPa to 200 kPa, for Specimen 2.](image2)
For the organic clay (Specimen 4), the consolidation curve shows high deformability and also high values for both $c_v$ and $c_{u}$. The almost absence of the inflexion point in this last curve should be noted. This fact makes the application of the standard graphical methods to the analysis very difficult, specially the Casagrande's method. But the regression method is capable of dealing with this type of situation without much trouble.

The application of the proposed method to the analysis of the other load increments, not shown here for brevity, presented similar results. Figures 5–8 show the variation of $c_v$ as a function of the applied vertical effective stress $\sigma'_{v}$ for the four specimens, calculated for all the load increments by the two standard manual methods and by the automatic least squares regression method. It can be seen that the results obtained by the three methods are not far from each other, with exception of the $c_v$ values obtained for the organic clay. For that soil, the differences between the $c_v$ values obtained with the three methods as well as the $c_v$ variation with the applied vertical stress is larger than for the other specimens. In all cases, however, the regression method gives results that are in the range of variation of the values obtained by the standard methods.

The magnitude of the coefficient of secondary compression $c_u$ is also obtained with the proposed method. Figure 9 shows the variation of $c_u$ with the vertical effective stress $\sigma'_{v}$ for the four specimens.

The plot in Fig. 9 shows that $c_u$ is heavily dependent on the soil type, but is less dependent on the applied load magnitude. As expected, the larger values of $c_u$ were obtained for the organic clay, whereas the smaller values were obtained for the low plasticity clay. Also, the effect of the bentonite content on $c_u$ can be clearly observed in the plot, increasing $c_u$ by a large amount. It should be
noted, however, that the obtained \( c_v \) values, specially those values for Sample 3, which has the lowest coefficient of consolidation, are not completely trustful, since they are based on a small number of readings in the secondary compression range of the consolidation curve. Long-term consolidation tests would be necessary to get higher confidence on the \( c_v \) values. Anyway, the obtained \( c_v \) values for all soils are in the range reported in the literature for those types of soil (Mesri and Godlewski 1977).

As stated in the previous section, the model beginning of secondary compression should be arbitrarily set in the proposed method. For the numerical results shown hereto, the model beginning of secondary compression is set to 95 % of the primary consolidation \( T_{95\text{os}} \). This setting has some effect on the obtained values of \( c_v \), depending on the amount of secondary compression in the soil behavior. Figure 10 shows the variation of the \( c_v \) values obtained for the load increment from \( \sigma'_v = 100 \) kPa to 200 kPa with the setting of the beginning of secondary compression, for all samples.

The increment of the calculated \( c_v \) values with the reduction of \( t_0 \), shown in Fig. 10, is much larger for the organic clay than for the other three samples, due to the larger amount of secondary compression in the organic clay settlement. But the earlier values of \( t_0 \) used to get the results in Fig. 10 are hardly justified, without additional information on the ongoing secondary compression, even taking the reports of beginning of secondary compression, based on experimental observations, at points prior to 75 % of primary consolidation for peat (Robinson 2003), into account. The linear-log \( t \) model used here probably does not apply to those reported, extreme cases, which require a more refined secondary compression model to match the experimental observations. Also, those early beginning of secondary compression times were observed in consolidation tests with load increment ratio much lower than 1. Thus, in order to stay consistent with the standard graphical methods, values between 90 and 95 % are suggested for the model beginning of secondary compression, for all soil types, when \( LIR = 1 \) and the linear-log \( t \) model is used, unless more information on the secondary compression for that soil type is available.

It is interesting to note, also, that the regression curves resulting from different settings of the model beginning of secondary compression are close to each other. Figure 11 shows the regression curves for Specimen 4, obtained by setting \( t_0 \) at 50 %, 75 %, and at
when the coefficient of secondary compression is high, as for soft kink, which is an artifact of the regression model, is apparent only /H20849 /H20849 /H20849 /H20849 /H20849 3 1.81 1.82 1.79 0.20 2 5.72 5.80 5.94 0.87 /H20849 /H20849 Specimen Original Set Reduced Set Mean Std. Dev. standard deviation values of the was repeated ten times for each reduced dataset. Then, mean and regression was applied to the resulting datasets. This second step dial gage face) was added to that reduced settlement values and the tic standard deviation of 0.02 mm (two divisions on the mechanical gage coupled to an automatic data logging system. In many cases, mechanical dial gage extensometers with 0.01 mm precision and gage coupled to an automatic data logging system. In many cases, mechanical dial gage extensometers with 0.01 mm precision and manual data logging are used in practice. In those cases, a much reduced number of settlement readings are taken, usually at times 8, 15, and 30 s, 1, 2, 4, 8, 15, and 30 min, 1, 2, 4, 8, and 24 h. Moreover, the manual data logging can lead to even less precision in the readings, specially for the initial ones, when the dial pointer is moving faster.

In order to test the proposed method behavior in face of those adverse conditions, a number of numerical simulations were made. The simulation process was applied in two steps. First, the extra readings between those taken at times close to those listed above were deleted from the datasets and the kept settlement values were rounded to two decimal figures. New values of \( c_s \) were then obtained with the regression method from these reduced datasets. Next, in the second step, a Gaussian random noise with a pessimistic standard deviation of 0.02 mm (two divisions on the mechanical dial gage face) was added to that reduced settlement values and the regression was applied to the resulting datasets. This second step was repeated ten times for each reduced dataset. Then, mean and standard deviation values of the \( c_s \) were calculated. Table 3 shows the results obtained with those simulations.

The numerical results in Table 3 show that the regression method works very well with the datasets with reduced number of points and lower precision, since the \( c_s \) values obtained with both the original and reduced datasets are very close to each other for all four specimens. Also, the mean \( c_s \) values obtained with the reduced set with added noise are close to the original \( c_s \) values. The observed relative standard deviation magnitude can be considered low, given the magnitude of the Gaussian noise introduced. As expected, the relative standard deviation magnitude is higher for the stiffer specimens, specially for Specimen 1, because the noise amplitude relative to the total settlement is higher for those specimens.

It is concluded, from the simulations made, that the proposed regression method is very stable and gives consistent results. Additionally, extensions to the regression method proposed herein can be devised. One can assign different weights to the readings, thus favoring some of them over the others. For example, larger weights can be assigned to the initial readings, which are less affected by secondary compression. In this way the resulting \( c_s \) will be less affected by the secondary compression. This weighted regression method can be implemented by replacing the least square procedure in Eq 8 with a weighted least square procedure:

\[
\min \sum_{i=1}^{N} w_i [\delta(t_i) - \delta]^2
\]

where \( w_i \) are the weights assigned to each reading.

Another possible development is to perform a compound simultaneous regression with datasets from more than one load increment at once, sharing the same \( c_s \) (and possibly the same \( c_o \)) (Bowen and Jerman 1995). This procedure will result in an average value for \( c_s \) (and \( c_o \)) that is appropriate for a load range wider than one single load increment.

### Concluding Remarks

A nonlinear regression method based on the least squares procedure for the analysis of the oedometer consolidation test data was presented. The method can deal with test data from different types of clay soils and calculates the values of the coefficient of consolidation \( c_s \) and of secondary compression \( c_o \) without the need of hand drawing the consolidation curve and of other graphical constructions. The resulting \( c_s \) and \( c_o \) obtained with this method have statistical significance since the least squares procedure is used.

The regression method can be implemented in spreadsheet computer programs that is commonly found in most soil laboratories. This implementation can be done with macro-programming or with the spreadsheet built in functions. In the last case the “goal seek” facility found in those computer programs can be used.

Numerical values for \( c_s \), from consolidation tests executed on different types of clays, show that the values obtained by the proposed method are in good agreement with those obtained by the standard graphical methods. Also, the values of \( c_o \) obtained with the proposed method are in the range of values reported in the literature.

Simulations through variation of data quality showed that the numerical regression procedure is very stable and leads to consistent results for most cases. For the rare cases that have convergence problems, a simple workaround can be applied.

The main limitation of the proposed method is the time of model beginning of secondary compression which has to be arbitrarily set at some degree of the primary consolidation. This limitation is common to all analysis methods based only on settlement measurements.

Possible extensions of the proposed method include the use of different models for the secondary compression, the use of a weighted least squares procedure, and the compound simultaneous...
regression, in which more than one load increment dataset is processed at once.

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Appendix: Algorithm Derivation for the Regression Method

The model assumed for the soil sample settlement in the oedometer, under a stress increment, as a function of the time is:

\[ \delta = \delta_0 + \delta_{100} U(\bar{\varepsilon}_v, t) + \bar{\varepsilon}_g S(t_0, t) \]  

where \( \delta_0 \) is the initial instantaneous displacement, \( \delta_{100} \) is the total primary consolidation displacement,

\[ U(\bar{\varepsilon}_v, t) = 1 - \frac{8}{\pi^2} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} e^{-\frac{(2m+1)^2 \pi^2 \bar{\varepsilon}_v^2}{4 t}} \]  

is the percentage of primary consolidation from Terzaghi’s theory, \( \bar{\varepsilon}_v = c_v \lambda / H_d \) is the modified coefficient of consolidation, \( H_d \) is the height of drainage, \( \bar{\varepsilon}_g = c_g H = c_s H / (1 + e_0) \) is the modified coefficient of secondary compression, \( H \) is the sample height, \( e_0 \) is the initial void ratio, and

\[ S(t_0, t) = \log_{10} \max \left( 1, \frac{t}{t_0} \right) \]  

where \( t_0 \) is the time of the model beginning of secondary compression. The parameters for the model above, to be determined from the measured settlement \( \delta_t \) at time \( t_i \) during the consolidation test, are \( \delta_0, \delta_{100}, \bar{\varepsilon}_v \), and \( \bar{\varepsilon}_g \). The time \( t_0 \) is supposed to be related to \( \bar{\varepsilon}_v \) by:

\[ t_0 = \frac{T_{BoS}}{\bar{\varepsilon}_v} \]  

where \( T_{BoS} \) is the time factor which corresponds to the model beginning of secondary compression, which is supposed to be known.

The regression algorithm is based on a least-squares fit procedure. The sum of the squared differences between the model displacements at \( t_i \) and the measured displacement \( d_i \) is:

\[ D = \sum_{i=1}^{N} R_i^2 \]  

where

\[ R_i = \delta_0 + \delta_{100} U(\bar{\varepsilon}_v, t_i) + \bar{\varepsilon}_g S(t_0, t_i) - \delta_i \]  

is the residue at the \( i \)th point.

Taking the derivatives of \( D \) relative to \( \delta_0, \delta_{100}, \) and \( \bar{\varepsilon}_g \), and equating each one to zero in order to get a minimum, a linear system of equations is obtained. This system, in matricial form, is:

\[ \begin{bmatrix} N & U(\bar{\varepsilon}_v, t_i) & S(t_0, t_i) \\ U(\bar{\varepsilon}_v, t_i) & U^2(\bar{\varepsilon}_v, t_i) & U(\bar{\varepsilon}_v, t_i) S(t_0, t_i) \end{bmatrix} \begin{bmatrix} \delta_0 \\ \delta_{100} \end{bmatrix} = \begin{bmatrix} \delta_i \\ \delta_i U(\bar{\varepsilon}_v, t_i) \end{bmatrix} \]  

(16)

with implied summation over \( i \).

A fourth equation is obtained by taking the derivative of \( D \) relative to \( \bar{\varepsilon}_v \) and equating it to zero:

\[ F = \frac{\partial D}{\partial \bar{\varepsilon}_v} = 2 \sum_{i=1}^{n} \left[ \delta_{100} U'(\bar{\varepsilon}_v, t_i) + \bar{\varepsilon}_g S'(\bar{\varepsilon}_v, t_i) \right] R_i = 0 \]  

(17)

where

\[ U'(\bar{\varepsilon}_v, t) = \frac{\partial U(\bar{\varepsilon}_v, t)}{\partial \bar{\varepsilon}_v} = \frac{2}{(2m+1)^2 \pi^2} e^{-\frac{(2m+1)^2 \pi^2 \bar{\varepsilon}_v^2}{4 t}} \]  

(18)

and

\[ S'(\bar{\varepsilon}_v, t) = \frac{\partial S(t_0, t)}{\partial \bar{\varepsilon}_v} = \frac{1}{\bar{\varepsilon}_v} \ln 10 \]  

(19)

The solution of the set of four equations is obtained by an iterative procedure based on a simple solution for the \( 3 \times 3 \) linear system and on the Newton-Raphson root-finding method for the last equation.

First, an initial guess \( \bar{\varepsilon}_v^{(0)} \) is estimated. The initial \( t_0 \) is then calculated from Eq 13 and \( \delta_0, \delta_{100}, \) and \( \bar{\varepsilon}_g \) are calculated from Eq 16. Next, a better approximation for \( \bar{\varepsilon}_v \) can be obtained from:

\[ \bar{\varepsilon}_v^{(k+1)} = \bar{\varepsilon}_v^{(k)} - \frac{F'}{F} \]  

(20)

where

\[ F' = \frac{\partial F}{\partial \bar{\varepsilon}_v} = 2 \sum_{i=1}^{n} \left[ \delta_{100} U''(\bar{\varepsilon}_v, t_i) + \bar{\varepsilon}_g S''(\bar{\varepsilon}_v, t_i) \right] R_i + 2 \sum_{i=1}^{n} \left[ \delta_{100} U'(\bar{\varepsilon}_v, t_i) + \bar{\varepsilon}_g S'(\bar{\varepsilon}_v, t_i) \right]^2 \]  

(21)

and

\[ U''(\bar{\varepsilon}_v, t) = \frac{\partial^2 U(\bar{\varepsilon}_v, t)}{\partial \bar{\varepsilon}_v^2} = -\frac{\pi^2}{2} \sum_{m=0}^{\infty} (2m+1)^2 e^{-\frac{(2m+1)^2 \pi^2 \bar{\varepsilon}_v^2}{4 t}} \]  

(22)

\[ S''(\bar{\varepsilon}_v, t) = \frac{\partial^2 S(t_0, t)}{\partial \bar{\varepsilon}_v^2} = -\frac{1}{\bar{\varepsilon}_v^2} \ln 10 \]  

(23)

The procedure is repeated until \( |\bar{\varepsilon}_v^{(k+1)} - \bar{\varepsilon}_v^{(k)}| \leq \epsilon \), where \( \epsilon \) is the required precision.

The initial guess \( \bar{\varepsilon}_v^{(0)} \) can be taken as:

\[ \bar{\varepsilon}_v^{(0)} = \frac{T_{BoS}}{t_{BoS}} 0.192 \]  

(24)

where \( t_{BoS} \) is the estimated time for 50 \% of the consolidation, which can be taken as the time that corresponds to half of the total settlement, excluding the initial settlement, due the load increment, \( \delta_{50} \).
\[ \delta_{50} = \frac{\delta_1 + \delta_N}{2} \]  

(25)

After calculating \( \delta_{50} \), \( \bar{t}_{50} \) may be estimated by linear interpolation from the two available readings closest to \( \delta_{50} \).

References


