DYNAMIC AXIAL RESPONSE OF SINGLE PILES EMBEDDED IN TRANVERSELY ISOTROPIC SOILS

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ABSTRACT

The response of piles with circular section embedded in transversely isotropic half spaces to time-harmonic axial loads is analyzed. The pile is modeled by the Finite Element technique as a series of bar elements. The soil is modeled by an indirect formulation of the Boundary Element Method. The BEM formulation uses influence functions which are displacements and stresses due to loads distributed along circular and cylindrical surfaces inside a transversely isotropic elastic half space. This approach has two main advantages: only the soil-pile interface needs discretization and the load sources can be applied directly along the true pile surface. The coupling between the two models is set at the middle point of the bar elements, rather than at the end points. This leads to a much more stable numeric solution and to a smoother load transfer profile. The results for the isotropic case are compared with numerical values obtained by other researchers. The influence of the soil anisotropy is addressed.

Keywords: Piles, Soil-structure interaction, Elastodynamics, Anisotropy

INTRODUCTION

Since the pioneering work of Novak (1977), the dynamic soil-pile interaction problem has been studied using numerical methods. Among these methods, the Boundary Element Method (BEM) has become standard for the soil modeling (Rajapakse and Shah 1987; Rajapakse and Shah, 1989). This approach, however, requires the previous knowledge of an auxiliary elastic state solution. Green’s functions for both full space and half space have been used in BEM formulations. The former provides formulations with more general character while formulations based on the latter avoid the free surface discretization.

When dealing with anisotropic media, the auxiliary elastic solution should also be anisotropic. For the case of transversely isotropic media, dynamic Green’s functions for both full and half spaces have been developed (Rajapakse and Wang 1993). These solutions, however, are expressed in terms of integrals which must be evaluated numerically as no analytical solution for that integrals is known. Since the BEM requires one further integration of the Green’s functions along the boundary elements, the computational cost of the formulations based on Green’s functions is excessively high.

In order to avoid the costs of the double numerical integration, the present work introduces a formulation for the BEM which is based on influence functions which are the elastic medium response to distributed loads. Influence functions for dynamic loads distributed along disks and cylindrical shafts and applied inside a transversely isotropic half space were derived. Also, since the influence functions present no singularities, it is possible to apply the loads on the true soil-pile interface as an additional benefit of the proposed method.

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FORMULATION

The objective of the present work is to obtain the steady-state response of the pile-soil system shown in the Fig. 1 to time-harmonic dynamic loads. The pile is vertical and cylindrical in shape with length \( l_p \) which is much greater than the pile radius \( a \). The soil is supposed to be an elastic, transversely isotropic half space in which the symmetry axis of elasticity is perpendicular to the free surface. The dynamic load is applied to the top of the pile in the vertical direction. A cylindrical coordinate system \( r\theta z \) is defined, so the problem is axisymmetric, with all variables independent of the \( \theta \) coordinate.

The pile is modeled by \( N \) linear displacement bar elements with nodes at the its ends. Constant radial \( t_r \) and vertical \( t_z \) tractions are applied along each element. The vertical external force \( F_z \) is applied at the first node on the pile top and radial \( t_r^{(N+1)} \) and vertical \( t_z^{(N+1)} \) are applied at the \((N+1)\)-th node at the pile tip. For the bar FEM model used, only the vertical tractions \( t_z^{(i)} \) are considered and the equation of motion of the pile nodes is expressed by:

\[
(K - \omega^2 M) \mathbf{u}_z + B t_z = \mathbf{f}
\]

where \( M \) and \( K \) are the mass and stiffness matrices, respectively (Weaver, Jr. and Johnston 1987). Also, \( \mathbf{u}_z \) is the vector of nodal vertical displacements, \( \mathbf{t}_z \) is the vector of vertical tractions \( t_z^{(i)} \) acting along the pile face and at the pile tip, \( B \) is the conversion matrix that translates the vertical tractions into equivalent nodal forces, \( \mathbf{f} \) is the vector of external applied loads and \( \omega \) is the circular frequency of the load.

The surrounding soil is modeled as an elastic, transversely isotropic half space. The stress-strain relationship for such medium, in cylindrical coordinates for axisymmetric strain, is given by:

\[
\sigma_{rr} = c_{11} \varepsilon_{rr} + c_{13} \varepsilon_{zz}
\]

\[
\sigma_{zz} = c_{13} \varepsilon_{rr} + c_{33} \varepsilon_{zz}
\]

\[
\sigma_{rz} = 2c_{44} \varepsilon_{rz}
\]

were \( c_{ij} \) are the material elastic constants.

Inside the half space, \( N \) radial and \( N \) vertical fictitious loads \( q_r^{(i)} \) and \( q_z^{(i)} \) uniformly distributed along cylindrical shafts with the same dimension of the pile bar elements are applied. Also, one
radial and one vertical fictitious disk loads $q_r^{(N+1)}$ and $q_z^{(N+1)}$ are applied at the pile base. The displacements and tractions along the soil-pile interface due to the fictitious loads are expressed by:

$$u_\alpha = \sum_{j=1}^{N+1} \left( U_r^{(j)} q_r^{(j)} + U_z^{(j)} q_z^{(j)} \right) \tag{5}$$

$$t_\alpha = \sum_{j=1}^{N+1} \left( T_r^{(j)} q_r^{(j)} + T_z^{(j)} q_z^{(j)} \right), \quad \alpha = r, z \tag{6}$$

where $U_{\alpha\beta}^{(j)}$ and $T_{\alpha\beta}^{(j)}$ are the influence functions for displacement and stresses respectively at the point at the interface due to a distributed unitary load in $\beta$ direction applied at the $j$-th element.

The influence functions were derived through analytical integration of the Green’s functions for transversely isotropic half spaces.

The coupling of the two models is accomplished by imposing displacement and traction compatibility on $N+1$ points along the soil-pile interface. Two different approaches were tried. The two approaches imposes traction compatibility at the center point of each element and at the half radius of the base.

In the first approach, the displacement compatibility is imposed at the nodes of the FEM model, which are located at the ends of each element. In this way, the displacement and traction vectors are expressed by:

$$u_r = U_{rr} q_r + U_{rz} q_z \tag{7}$$

$$u_z = U_{zr} q_r + U_{zz} q_z \tag{8}$$

$$t_r = T_{rr} q_r + T_{rz} q_z \tag{9}$$

$$t_z = T_{zr} q_r + T_{zz} q_z \tag{10}$$

were $U_{\alpha\beta}$ and $T_{\alpha\beta}$ are the influence matrices for displacements and tractions respectively. Using the pile equation of motion, the following system of equations is obtained:

$$\begin{align*}
(\bar{K}U_{zz} + BT_{zz}) q_z + (\bar{K}U_{rr} + BT_{rr}) q_r &= f \\
U_{rz} q_z + U_{rr} q_r &= 0
\end{align*} \tag{11}$$

were:

$$\bar{K} = K - \omega^2 M \tag{12}$$

The solution of this system of equations gives the fictitious loads $q_r^{(i)}$ and $q_z^{(i)}$ along the pile-soil interface. Using these values one can calculate the displacements and tractions along the interface and also at any point of the half-space.

The numerical performance of this first method, however, was found to be very poor. The resulting traction $t_z$ distribution along the interface results very irregular and the convergence of the displacements is very slow.

In order to improve the results, a second approach was tried. In this approach the displacement compatibility is imposed at the same points were the traction compatibility is imposed. At these points the displacements are expressed by:

$$\bar{u}_r = \bar{U}_{rr} q_r + \bar{U}_{rz} q_z \tag{13}$$

$$\bar{u}_z = \bar{U}_{zr} q_r + \bar{U}_{zz} q_z \tag{14}$$
Since the FEM model displacements nodes are located at the end of the elements, a conversion matrix \( L \) is introduced as:

\[
\bar{u}_z = Lu_z
\]

(15)

The elements of \( L \) are obtained from the FEM model bar element linear displacement interpolation functions as:

\[
\bar{u}_z^{(i)} = \frac{1}{2} \left( u_z^{(i)} + u_z^{(i+1)} \right), \quad i = 1, \ldots, N
\]

(16)

\[
\bar{u}_z^{(N+1)} = u_z^{(N+1)}
\]

(17)

The new system of equations results:

\[
(\bar{K}L^{-1}\bar{U}_{zz} + BT_{zz}) q_z + (\bar{K}L^{-1}\bar{U}_{zz} + BT_{zz}) q_r = f
\]

\[
\bar{U}_{rz} q_z + \bar{U}_{rr} q_r = 0
\]

(18)

The numerical results obtained by this last approach shows much smoother distribution of the vertical tractions along the pile and much better convergence of the displacement.

**NUMERICAL RESULTS**

For the purpose of analysis of the influence of the medium anisotropy degree on the pile response to vertical loads, two anisotropy indices \( n_1 \) and \( n_3 \) are used. They are defined as:

\[
n_1 = c_{33}/c_{11}
\]

(19)

\[
n_3 = (c_{11} - 2c_{44})/c_{13}
\]

(20)

The numerical results are expressed as vertical impedances \( K_{vv} \), defined as:

\[
K_{vv} = \frac{F_z}{u_z^{(1)}ac_{44}}
\]

(21)

and the non-dimensional frequency \( a_0 \), is defined by:

\[
a_0 = \omega a \sqrt{\frac{D_s}{c_{44}}}
\]

(22)

where \( \rho_s \) is the soil density.

The Table 1 presents some numerical values of the vertical impedance of a pile with \( l_p/a = 10 \) embedded in an elastic isotropic soil. The \( K_{vv} \) values obtained with the present formulation are compared with those obtained by Rajapakse and Shah (1987). As can be seen, the results obtained by the two methods are very close.

**TABLE 1. Comparison of \( K_{vv} \) values \( (l_p/a = 10, \rho_p/\rho_s = 1, \nu = 0.25, E_p/E_s = 1000) \)**

<table>
<thead>
<tr>
<th>( a_0 )</th>
<th>Present method</th>
<th>Rajapakse and Shah 1987</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real</td>
<td>Imag.</td>
</tr>
<tr>
<td>0.4</td>
<td>24.25</td>
<td>31.80</td>
</tr>
<tr>
<td>1.0</td>
<td>2.62</td>
<td>70.43</td>
</tr>
<tr>
<td>1.5</td>
<td>-33.72</td>
<td>105.7</td>
</tr>
</tbody>
</table>
The plots in Fig. 2 show the influence of $n_1$ and $n_3$ on the impedance $K_{vv}$ for a pile having a length ratio $l_p/a = 50$, density ratio $\rho_p/\rho_s = 1.5$ and Young’s modulus ratio $E_p/E_c = 1000$. The soil vertical Young’s modulus $E_z$ is given by:

$$E_z = c_{33} - \frac{c_{13}^2}{c_{11} - c_{44}} \tag{23}$$

For all cases, the soil has a ratio $c_{33}/c_{44} = 3$. For the isotropic case (when $n_1 = 1$ and $n_3 = 1$) this corresponds to a Poisson’s ratio $\nu = 0.25$.

The results in Fig. 2 show that an increase in $n_1$ leads to an increase in $K_{vv}$. A similar behavior is observed in the $n_3$ influence, but with less intensity. It is interesting to note that the curves for the cases $(n_1 = .5, n_3 = 1)$ and $(n_1 = 1, n_3 = .5)$ are almost coincident. This coincidence may not occur in other cases.
CONCLUDING REMARKS

The method of analysis proposed here is effective when dealing with the problem of a vertical pile embedded in a transversely isotropic soil. The numerical results show that the soil anisotropy has a marked influence on the pile response. It should be noted, also that when restricted to the isotropic case the results compares well with those published by other researchers.

The same approach can be used to access the response of piles subjected to torsion loads. In this case, influence functions for torsion loads are necessary. On the other hand, the pile response to transverse loads requires a more complex analysis due to the coupling between the horizontal displacement and the rotation. Even in this last case, however, the analysis can benefit from the present method.

A simplified version of the present method can be derived by neglecting the radial displacement and traction influence. This would lead to a system of equations with half the size of that of the original method. The impact of this simplification on the results should be evaluated.

ACKNOWLEDGMENT

The work presented here is supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP).

REFERENCES