



Probabilistic and Deterministic Analysis of an Excavation Supported by Tiebacks and Nailing in Residual Soil of Gneiss

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Abstract. With the growing economic appreciation of urban areas in large cities, building techniques that are technically and economically feasible have been widely employed to maximize utilization of spaces. Such techniques resort to large excavations to expand the land occupation rate. In this context, this paper analyzes a 24-m deep excavation located in São Bernardo do Campo, a city in the metropolitan region of São Paulo Metropolitan Region, Brazil. The local subsoil is composed by residual gneiss soil, and the retainment was executed in soil nailing. The length of the nails is variable in depth; the longer nails are introduced in the first 8 m of the excavation, and then reduced in greater depths. During the development of the retainment design, numerical analyses via limit equilibrium using deterministic and probabilistic methods considering the variations in properties of materials, demonstrated that the nails close to the end of the excavation are weakly influenced by the changes in nail lengths and diameters. In this sense, new analyses were performed to simulate the replacement of 1/3 of the nail rows for one or two rows of tiebacks at the end of the excavation to maintain stability of the retainment and reduce costs. The results for safety factor were obtained in deterministic and statistic (mean) terms. In the latter case, the reliability index and the probability of failure of the analyses performed are also presented. The results demonstrate the importance of conducting investigative analyses to determine the critical rupture surface by means of statistical analyses. In the case shown, this type of analysis proved to be more critical than the deterministic method of analysis of stability of retainment.

1 Introduction

The soil nailing technique has been widely used as a solution for retainment of large excavations in densely urbanized areas in several places worldwide, including Brazil. According to Liu, Shang, and Wu (2016), over the last decades soil nailing has become one of the most effective reinforcing methods, and it has been widely used in geotechnical structures such as excavation or slopes. In the first applications, soil nailing was considered as an alternative solution to conventional techniques to contain

excavations; it is now widely used in projects due to its acceptance and efficacy (Thomas 2003, 1997; Chassie 1993). According to Ghareh (2015), the advantage of soil nailing is its low cost, easy execution, consolidated technique and shorter time of execution, which has led to it being extensively used in engineering.

Technically, the nails prevent both soil strain and failure of the massif by transferring uplift forces generated by shear forces to the entire length of the inclusion (Luo, Tan, and Yong 2000). As a rule, the traditional method of safety coefficient is used to analyze stability of soil nailing. This method is based on the theory of limit equilibrium (Ugai and Leshchinsky 1995; Thomas 1997; Turner and Jensen 2005). However, retainment in nailed soil should be projected in order to be safety for all potential modes of rupture, including those caused by external and internal means and also on the wall surface. External ruptures refer to failures in global stability, sliding and support capacity at the bottom. In this type of analysis, the characteristics of the soil and of the nails are considered as homogeneous and all unknown, uncertain factors are attributed to a single significant coefficient.

According to Liu, Shang, and Wu (2016), there are uncertainties about these parameters: rupture may occur even if the calculated safety coefficient (FS) is greater than the minimum (FS_{\min}) in conformance with the safety codes of the different countries. Therefore, it is important to carry out analyses on soil nailing stability that consider variability of the parameter of the soil and of the nails. In this sense, it becomes important to use ruin probability analysis to assess excavation stability (Babu and Singh 2009; Wright and Duncan 1991). Several studies have shown results on the variability of these parameters and the confidence intervals to be used in the analyses. According to Griffiths and Fenton (2004), the quick advances in IT have made it easier to use numerical methods that employ finite elements or finite differences methods in the retainment analyses.

In general, the analyses used routinely only check excavations in which the soil is considered as homogeneous, with permanent geotechnical parameters for a determined layer of soil. Therefore, it is necessary to better understand the behavior of retainments using the resource of probabilistic analyses instead of the analyses carried out conventionally via deterministic means. Therefore, this work has been conceived to analyze the safety of a retainment on nailed soil by means of ruin probabilistic analysis.

1.1 Safety Factors

The stability analyses of excavations can be performed via determination of two types of significant factors (FS), either via deterministic or probabilistic means. The deterministic safety factor is calculated for the minimum global sliding surface, from the stability analysis of regular slope (non-probabilistic). In this type of analysis, all input parameters are exactly like the mean values. In the probabilistic analysis, the mean safety factor is obtained, which is the mean value of all safety factors calculated for the minimum global sliding surface. In general, the mean safety factor should be close to the value of the deterministic safety factor. For a sufficiently large number of samples, the two values should be close; however, this does not take place in practice, since the samples analyzed have limited quantities and sometimes are not representative of the subsoil analyzed.

1.2 Variations in Soil Parameters

Significant variability is associated to soils and rocks in comparison to other engineering materials such as steel or concrete. Such variability increases the risk associated to forecasts (Ameratunga, Sivakugan, and Das 2016). Table 1 summarizes the values for coefficient of variation of some geotechnical parameters reported in the literature (Duncan 2000; Sivakugan, Arulrajah, and Bo 2011). In this sense, the variation coefficient is defined as:

Table 1. Typical values of the variation coefficient of some soil parameters (modified from Ameratunga, Sivakugan, and Das 2016)

Parameter	Coefficient of variation (%)
Unit weight, γ	3–7
Bouyant unit weight, γ'	0–10
Friction angle (sands), ϕ'	10
Friction angle (clays), ϕ'	10–50
Undrained shear strength, c_u	20–40
Standard penetration test blown count, N	20–40

$$CV(\%) = \frac{S}{\bar{X}} \cdot 100 \tag{1}$$

Where: CV is the variation coefficient, S is the standard deviation and \bar{X} is the mean.

For the analysis, the relative minimum and maximum values are converted into real minimum and maximum values when the statistic sampling is made for each random variable, as follows:

1.3 Types of Analysis

In this work, two types of analyses were conducted: the “minimum global analysis”, which is performed via sampling method (Monte Carlo) from 1,000 samples. The minimum global is analyzed, i.e., the probabilistic analysis is conducted on the minimum global sliding surface located by the regular stability analysis (deterministic). This way, the safety factor is re-dimensioned N times (where N is the number of samples) for the “minimum global” sliding surface, using a different set of input variables randomly generated for each analysis.

1.4 Probability of Rupture and Reliability Index

The probability of failure is simply equal to the number of analyses with safety factor smaller than 1, divided by the total number of samples.

$$PF(\%) = \frac{N_{ruptura}}{N_{analises}} \cdot 100 \tag{2}$$

Where: $N_{ruptura}$ is the number of samples analyzed that produced a FS lower than 1, and $N_{analises}$ is the total number of samples analyzed, in this case equal to 1,000.

The Reliability Index is another commonly used measurement of stability of slopes, after a probabilistic analysis is performed. The reliability index is an indication of the number of standard deviations that separate the mean safety factor from the critical safety factor (=1). The Reliability Factor can be calculated by assuming a normal distribution (Eq. 3) or a lognormal distribution (Eq. 4) of the results of the safety factor. The reliability index is usually equal to 3, which is the minimum recommended for safety of the stability project.

$$\beta_N = \frac{\mu_{FS} - 1}{\sigma_{FS}} \tag{3}$$

Where: β_N is the reliability index, μ_{FS} is the mean of the safety factor, and σ_{FS} is the standard deviation of the safety factor.

$$\beta_{LN} = \frac{\ln\left[\frac{\mu_{FS}}{\sqrt{1+V^2}}\right]}{\sqrt{\ln(1+V^2)}} \tag{4}$$

Where: β_{LN} is the reliability index, μ_{FS} is the mean of the safety factor, and V is the variation coefficient of the safety factor ($= \sigma/\mu$).

1.5 Resistance of the Soil / Nail Contact

The friction strength between the soil and the nail was determined according to the experience in Brazilian soils, from back analysis of pullout tests (Fig. 1). The Eq. (5) below is based on the results of the simple sounding (N_{SPT}), per Eq. 3.

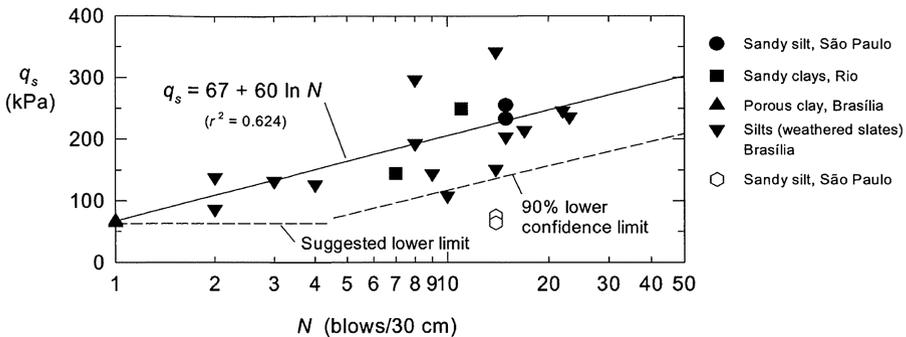


Fig. 1. Pullout tests results (Ortigão and Sayao 2004)

$$q_s = 67 + 60 \cdot \ln(N_{SPT}) \quad (5)$$

The allowable friction stress used in the design was determined for a safety factor equal to 2, per Eq. 6:

$$q_{adm} = \frac{q_s}{FS} (FS \geq 2) \quad (6)$$

1.6 Materials and Methods

The excavation area is located downtown São Bernardo do Campo city/SP, designed for construction of a 15-floor apartment building with three underground floors. The excavation was executed including retainment of its sides in nailed soil, totaling 1,747 m² of sprayed concrete, 2,172 nails and 267 deep sub-horizontal drains (Cut 2017).

The nails were perforated with a 0.12 m diameter. The nails were previously assembled and then installed in the perforations. The nails included four sectors of injection besides the sheath (bore filling). The steel bars were of the CA-50 type. They were guarded with epoxy paint and the splices were made with steel sleeves pressed with a hydraulic device. The nails were filled and injected with cement slurry with a water/cement factor of 0.5 of weight. For the eight upper rows of nails, bars with a 20-mm diameter were used; in the eight center rows, the bars had a 25-mm diameter, and the eight lower lines had a 32-mm diameter.

The wall surface of sprayed concrete with thickness in the order of 0.10 m included an electro-welded metal mesh.

Wall drains (strips of geocomposite placed between the soil / sprayed concrete contact) and deep sub-horizontal split in six lines along the height of the retainment were used as the draining system.

The local subsoil is composed of gneiss residual soil and the retainment was executed on nailed soil. From the results of the investigations and subsoil characterization tests, the schematic profile was obtained with the key information on retainment, quantity and length of nails besides geological and geotechnical characteristics of the soil (Fig. 2).

1.7 Results and Analysis

Three potential failure surfaces were analyzed. These surfaces were established per recommendations from the geotechnical literature to check stability of nailed retainments and tieback retainments. Three potential failures surfaces were analyzed as determined per recommendations in the geotechnical literature to check stability of nailed and tieback retainments. (Figure 3). In the situations under analysis, the deterministic safety factor was determined by the Ordinary method (Fellenius).

After pre-defining the failure geometries (Fig. 3) and by means of the Slide 7.0® program (Rocscience), using the ruin analysis tool, it was possible to get the safety factors, probability of failure and the normal and lognormal reliability indexes. The

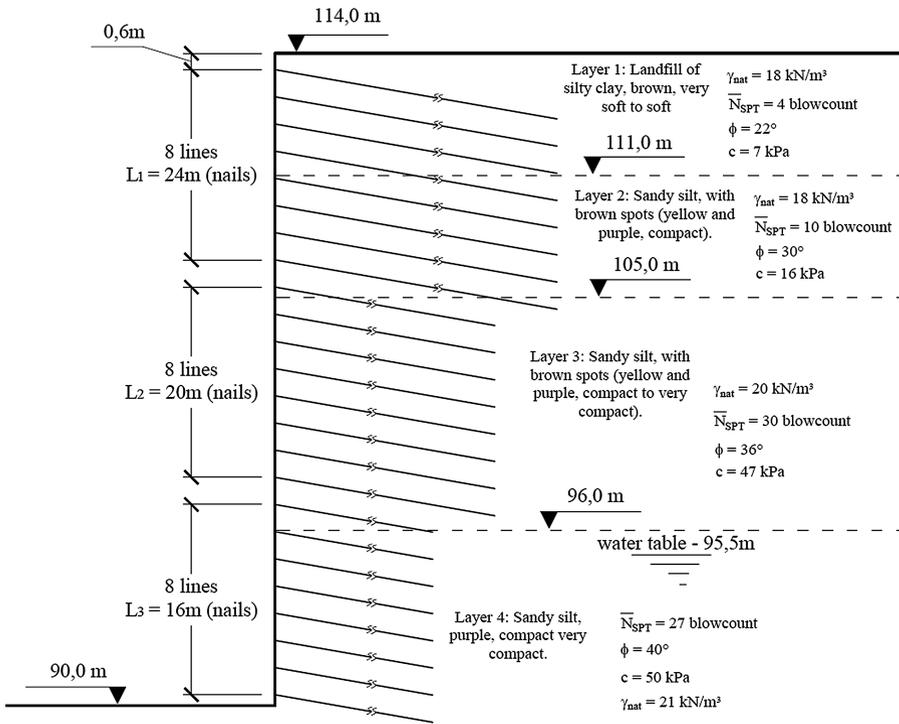


Fig. 2. Schematic profile of the retaining and soil properties

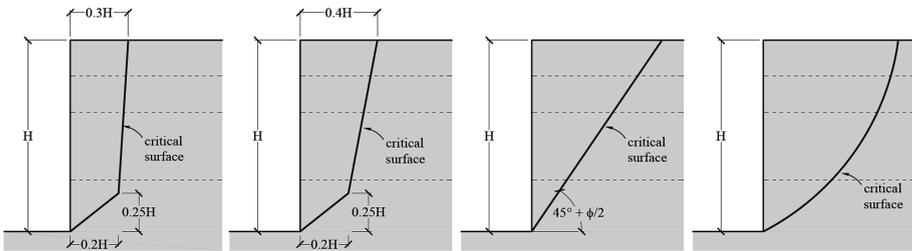


Fig. 3. Types of critical surfaces under analysis

analyses considered the failure surfaces indicated in Fig. 3 and several reinforcement conformations, as follows: 24 rows of nails (Figs. 4, 5, 6 and 13), 16 rows of nails and one row of tiebacks (Figs. 7, 8, 9 and 14), and 16 rows of nails and two rows of tiebacks (Figs. 10, 11, 12 and 15). For purposes of analysis, we checked the effect of decreasing the rows of nails from 24, and replacing for a tieback in one case, and by two tiebacks in another case.

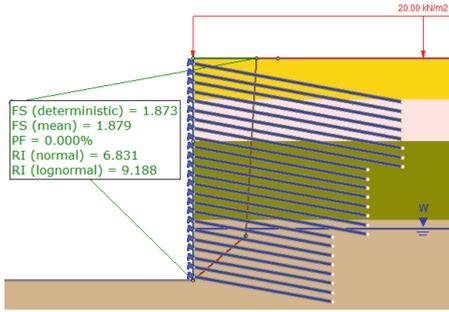


Fig. 4. Rupture surface in 0.3 H for 24 rows of nails

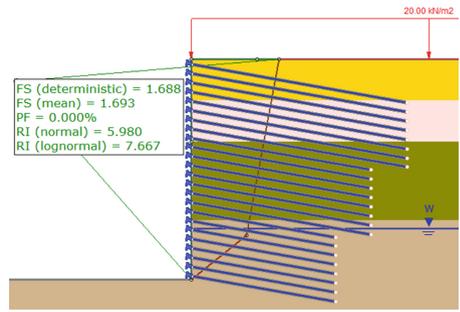


Fig. 5. Rupture surface in 0.4 H for 24 rows of nails

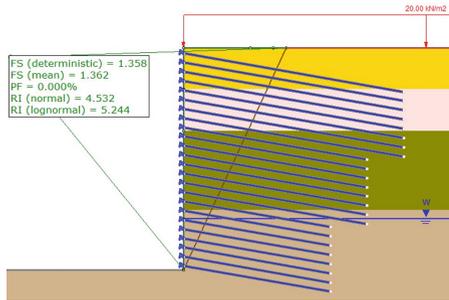


Fig. 6. Wedge-shaped rupture surface for 24 rows of nails

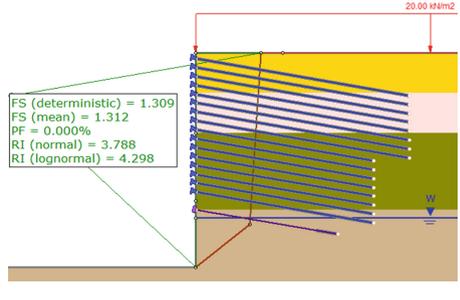


Fig. 7. Rupture surface in 0.3 H for 16 rows of nails and 1 row of tiebacks

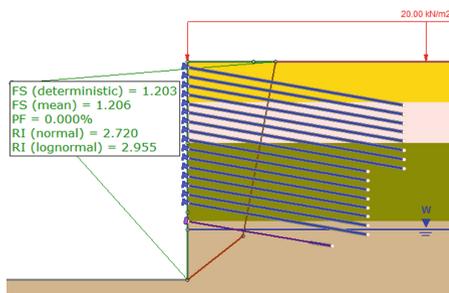


Fig. 8. Rupture surface in 0.4 H for 16 rows of nails and 1 row of tiebacks

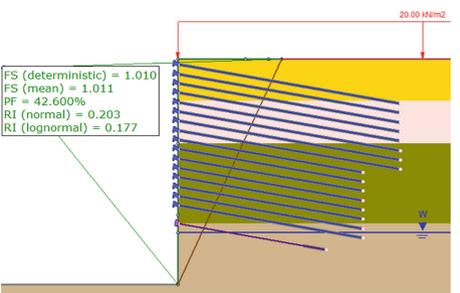


Fig. 9. Wedge-shaped rupture surface for 16 rows of nails and 1 row of tiebacks

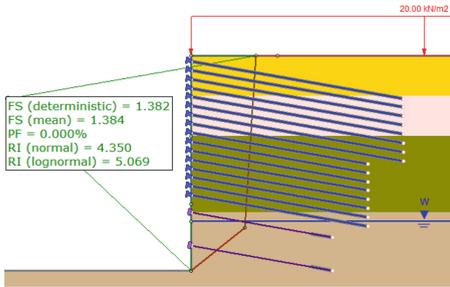


Fig. 10. Rupture surface in 0.3 H for 16 rows of nails and 2 rows of tiebacks.

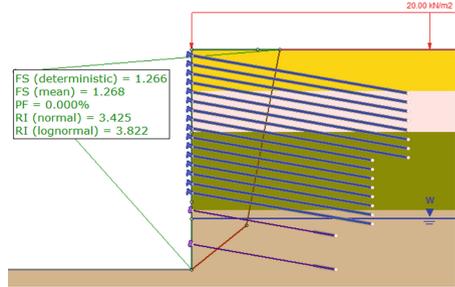


Fig. 11. Rupture surface in 0.4 H for 16 rows of nails and 2 rows of tiebacks

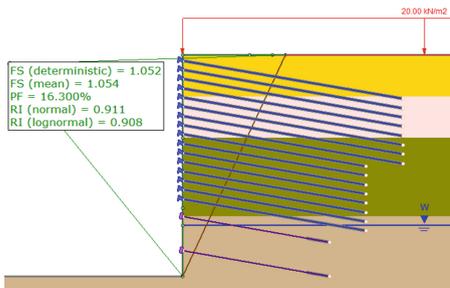


Fig. 12. Wedge-shaped rupture surface for 6 rows of nails and 2 rows of tiebacks.

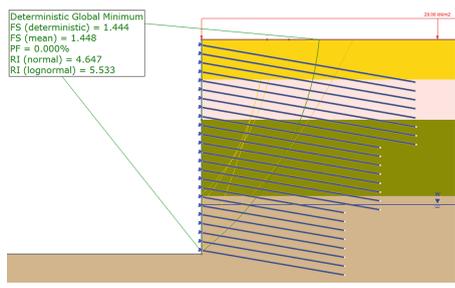


Fig. 13. General rupture for 24 rows of nails

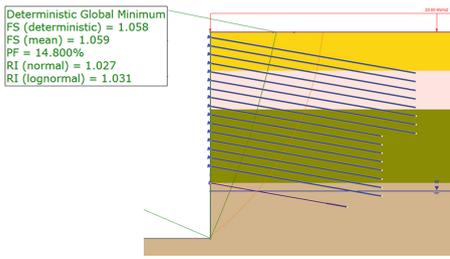


Fig. 14. General rupture for 16 rows of nails and 1 row of tiebacks

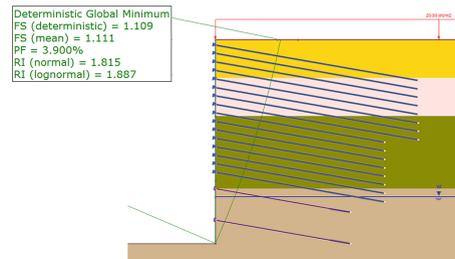


Fig. 15. General rupture for 6 rows of nails and 2 rows of tiebacks.

The results described in Table 3 show that, in all analyses made, the deterministic and mean (probabilistic) safety factors did not display significant differences, thus demonstrating that the number of samples analyzed can be considered appropriate.

The variability imposed to the soil parameters made it evident that the stability of the retainment was close to rupture ($FS = 1$) only for wedge-shaped rupture ($45^\circ + \phi/2$) (Figs. 9 and 12), with probability of failure at 42.6% and 16.3%, considering

replacement of the last eight rows of nails for one and two rows of tiebacks respectively. However, in all analyses made for this case, the probability of failure was observed, since the reliability index was lower than the minimum ($\beta = 3$), which demonstrates the potential for failures. Por outro lado, as análises em termo de ruptura geral são satisfatórias ($FS \approx 1.45$ e $\beta > 3$) somente para reforço integralmente efetuado com solo grampeado. For the option of replacing the last 6 nail lines by one and two tiebacks, the results of FS were 1.05 and 1.11. The probability of failure at 14.8% and 3.9%, respectively (Table 2).

Table 2. Soil resistance parameters

Layer	Interval	Young modulus	Cohesion	Friction angle	Specific weight
[-]	[m]	[kPa]	[kPa]	[°]	[kN/m ³]
1	114–111	3,300	7	22	18
2	111–105	6,000	16	30	18
3	105–96	20,500	47	36	20
4	96–86	55,000	50	40	21

The values obtained for safety factors, ruin probability and reliability index for each situation can be seen in Table 3.

Table 3. Summary of the results obtained from the analyses

Surface	Reinforcement	FS _{det}	FS _{mean}	PF (%)	RI _(normal)	RI _(lognormal)
0.3 H	24 rows of nails	1.873	1.688	0.000	6.831	9.188
0.4 H		1.879	1.693	0.000	5.980	7.667
45° + $\phi/2$		1.358	1.362	0.000	4.532	5.244
Global		1.444	1.448	0.000	4.647	5.533
0.3 H	16 rows of nails + 1 tieback	1.309	1.312	0.000	3.788	4.298
0.4 H		1.203	1.206	0.000	2.720	2.955
45° + $\phi/2$		1.010	1.011	42.600	0.203	0.177
Global		1.058	1.059	14.800	1.027	1.031
0.3 H	16 rows of nails + 2 tiebacks	1.308	1.384	0.000	4.350	5.069
0.4 H		1.266	1.268	0.000	3.425	3.822
45° + $\phi/2$		1.052	1.054	16.300	0.911	0.908
Global		1.109	1.111	3.900	1.815	1.887

Replacing eight rows of nails close to the end of the excavation (elevation 90 m) for one row of tiebacks significantly reduced the safety factor ($FS \cong 1$). This can be interpreted as a probable risk to retainment stability. On the other hand, when a second row of tiebacks was added, in the case of the critical wedge-shaped surface, the observation was that the safety factor obtained remained unchanged and equal to 1.0, close to rupture.

An analysis of the rupture of the “face”, for the distance of 0.3 H and 0.4 H upstream, showed that the option with two rows of tiebacks displayed a somewhat

higher safety factor (Figs. 10 and 11) in comparison to the option of one row of tiebacks (Figs. 8 and 9), as an alternative to replacing the eight rows of nails close to the end of the excavation.

2 Conclusions

In general, deterministic analyses are used to analyze stability of retainment / excavations. In this case, the intrinsic variabilities of soil properties are not assessed. However, in some cases, they can point out factors of acceptable global safety (deterministic), but with values of ruin probability that indicate high risk of the work.

For the three situations of rupture reviewed in this paper, (0.3 H, 0.4 H and $45^\circ + \phi/2$), the conclusion was that the situations composed fully by nails (24 rows) displayed safety factors above 1.5 and a reliability index above 3.0. This demonstrates that, for this solution of retainment, the risk is either minimal or nonexistent.

In the case of replacement of eight rows of nails for one or two rows of tiebacks, the safety factor dropped to values below 1.5, to show a negative effect on the safety of the work. This fact was corroborated mainly via the reliability index below 3.0 in the cases of rupture surfaces 0.4 H and $45^\circ + \phi/2$. It must be pointed out that, in these cases, the probability of ruin is 42.6% and 16.3% for one and two rows of tiebacks respectively in the case of surface at $45^\circ + \phi/2$. This indicates that, for the situation under analysis of 1,000 samples, 426 (one tieback) and 163 (two tiebacks) pose the risk of ruin. The analysis of the global failure was strongly influenced by the replacement of nail lines for tiebacks in the critical region near the end of the excavation.

The results obtained demonstrated the importance of conducting investigative analyses to determine the critical rupture surface by means of deterministic and probabilistic analyses. In the case under study, this type of analysis proved to be more critical than the deterministic method of analysis of stability of retainments since, besides the safety factor, values of probability of ruin and reliability index of the analyses can be obtained by means of variability of soil parameters.

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